DIFFY TOWERS:
EXPLORING SUBTRACTION AS COMPARISON

Matt Sexton, Australian Catholic University

Diffy Towers has been part of my practice for teaching subtraction for 20 years. I still use it in my university work today. Diffy Towers supports students in understanding the comparison (or difference) interpretation of subtraction.

Figure 1. Students engaging in a discussion convincing each other that the difference calculation is correct.

DIFFERENCE AND SUBTRACTION TYPES

Mathematically, difference is the value obtained by subtraction (Nelson, 2008), and the general name of the outcome (answer) of subtraction (Rowland, 2004). Difference is also a term we use to name a type of subtraction. Schell and Burns (1962) claimed that subtraction was multi-faceted, and that there were three basic types of subtraction: subtraction as 'take away'; subtraction as 'how many more needed'; and, subtraction as 'comparison or difference'. Carpenter, Fennema, Franke, Levi, and Empson (1999) later identified subtraction problem types as: subtraction as 'separation'; subtraction as 'part-part-whole'; and, subtraction as 'comparison'.

SUBTRACTION AS COMPARISON

Subtraction as comparison or difference involves thinking about the relationship between quantities as opposed to a separating action associated with subtraction as take away (Carpenter et al., 1999). When thinking about subtraction in this way, we consider two distinct, disjoint quantities at the same time and compare their sizes (Carpenter et al., 1999; Rowland, 2004). Schell and Burns (1962) claimed that when calculating the answer to a subtraction as comparison problem, we match the quantities and then identify the excess of the first quantity over the second quantity. This excess is considered the difference. The following is an example of a problem situation highlighting subtraction as comparison:

> *Michelle made a Unifix tower of 12 blocks. Jill made a tower of 18 Unifix blocks. Jill had how many more blocks than Michelle?*

In this example, one set, Michelle's 12 Unifix blocks, acts as the 'referent set'. It is compared to the other set, Jill's tower of 18 Unifix blocks, acting as the 'compared set'. The amount by which one set exceeds the other is considered the difference.

Rowland (2004) suggested that we focus on using 'how many more?' and 'how many less?' questions when asking students to compare the size of numbers. Rowland warned that focusing just on the term 'difference' can be difficult for young children to understand, so he suggested that teachers use the term 'compare' (and its derivatives, e.g., 'comparison' 'comparing') for greater clarity.

This advice from Rowland (2004) is important. In my use of Diffy Towers, when asked to talk about the difference between two numbers, students have often provided responses like 'one is even, one is odd', 'one is written this way, and the other that way (gesturing with fingers), as well as 'one is prime and the other is not'. Only after several opportunities of playing do we use the term 'difference', and we then talk about why the game might be called 'Diffy Towers'.

DIFFY TOWERS

One important mathematical idea we need to attend to is concerned with difference meaning *absolute difference*. In this sense, the difference between 5 and 3 is 2, and the difference between 3 and 5 is 2. Difference, therefore, is a positive number and also includes zero.

Year level: Foundation – Year 2.

Mathematics: Subtraction as comparison, subtraction counting and reasoning strategies, subtraction basic facts.

Materials: Unifix blocks; two regular, six-sided dice.

© The Mathematical Association of Victoria

Group setting: Pairs.

Instructions:

- Partner A rolls two dice and names the numbers that are rolled.
- Partner A calculates the difference between the numbers using an appropriate strategy and/or knowledge, and explains how they know the difference.
- Partner A convinces Partner B that their thinking is correct. If consensus is reached, Partner A takes the number of Unifix blocks that is equal to the difference and begins building their 'diffy tower'. For example, Partner A has rolled 4 and 6 and works out that the difference is 2. After convincing Partner B, Partner A takes 2 Unifix blocks to start their tower (see Figure 1).
- Repeat, swapping roles with a focus on students convincing each other that their calculations for difference are appropriate including the strategies and/or knowledge used to calculate the difference. Encourage use of the phrase 'Convince me!' to focus on the development of convincing arguments.
- After an appropriate amount of playing time, players check who has created the highest tower, and then identify the difference between the two towers.

One important pedagogical idea concerns the teacher's press on students convincing their partner that they have successfully identified the difference between the numbers rolled using the dice. Elements of the convincing process require naming the difference, describing the strategies or knowledge used to identify the difference, and providing convincing reasons for knowing the difference. In Figure 1, two students are seen engaging in a discussion before taking the number of Unifix blocks that matches the difference.

I always highlight the importance of students talking about their reasoning, i.e., what they know and how they know.

LINKS TO AUSTRALIAN CURRICULUM

Foundation

- Subitise small collections of objects. (ACMNA003)
- Compare and make correspondences between collections, initially to 20, and explain reasoning. (ACMNA289)
- Use direct comparisons to decide which is longer… and explain reasoning in everyday language. (ACMMG006)

Year 1

- Represent and solve simple subtraction problems using a range of strategies. (ACMNA015)

Year 2

- Explore the connection between addition and subtraction. (ACMNA029)
- Solve simple subtraction problems using a range of efficient mental strategies. (ACMNA030)

FROM COUNTING STRATEGIES TO AUTOMATIC RECALL

Diffy Towers can support fluency with the subtraction basic facts. It is important to note though that automatic response is the final phase of mastering the basic facts (Baroody, 2006). Before mastery, students need time to explore using counting strategies, and then move to exploring and using reasoning strategies. After appropriate time and reflection, students can begin recalling answers to basic facts.

TYPICAL COUNTING STRATEGIES INITIALLY USED

- representing each number rolled on the dice by making 'mini towers' with Unifix blocks, and directly comparing lengths of the towers to see the difference
- matching pips on each dice, then counting how many more pips on the other dice, and explaining how those other pips represent the difference (see Figure 2) and,
- using subtraction counting strategies: 'count back' (e.g., if 6 and 4 are rolled, start at 6 and count back 4, stopping at 2, knowing the final number counted is equal to the difference); 'count down to' (e.g., if 5 and 3 are rolled, start at 5 and count down to 3, knowing the number of counts is equal to the difference); and, 'count up to' (e.g., if 2 and 4 are rolled, start at 2 and count up to 4, knowing the number of counts equals the difference).

Figure 2. Matching the number of pips on the dice, then noticing how many more are on the other dice to identify the difference.

When supported by effective instruction, students typically begin to use reasoning strategies. This happens when students notice number relationships and make sense of number combination knowledge (Baroody, 2006). This noticing is facilitated through teacher questioning and when the students start to engage

© The Mathematical Association of Victoria

each other in purposeful talk focusing on explaining, elaborating, and convincing.

TYPICAL REASONING STRATEGIES USED

- 'one more/one less' principle (e.g., if 3 and 4 are rolled, knowing that 4 is one more than 3, so therefore the difference is 1).
- 'two more/two less' principle (e.g., if 1 and 3 are rolled, knowing that 1 is two less than 3, so therefore the difference is 2).
- doubles facts (e.g., if 3 and 6 are rolled, knowing that double 3 is 6, therefore the difference is 3).
- number combinations (e.g., if 5 and 1 are rolled, knowing that 1 and 4 are parts of 5, so therefore the difference is 4) and,
- 'think addition' or recognising the link between subtraction and addition (e.g., if 2 and 5 are rolled, think what number when added to 2 is equal to 5, therefore the difference is 3).

When students are prompted to think more about number combinations and knowledge, they can begin to develop knowledge of basic facts. It is important to realise is that it takes many opportunities for younger students to develop this bank of known facts (Baroody, 2006).

SUPPORTING CONVINCING USING SENTENCE FRAMES

The Australian Curriculum: Mathematics proficiencies are just as important as the content descriptors when it comes to teaching and learning mathematics. All proficiencies (Understanding, Fluency, Problem Solving, and Reasoning) are equally important, and language plays an important role during the enactment of these proficiencies. Language is vital if we want students to explain, elaborate, and extend their thinking to convince others of their mathematical reasoning. One easy way of supporting students to compose convincing arguments and focus their use of language structures is through the use of sentence frames (Bresser, Melanese, & Sphar, 2009).

A sentence frame is usually a sentence (or sentences) with some words removed, acting as a type of cloze activity. When using the frames, students choose appropriate vocabulary and complete the sentences using information about their thinking (Bresser et al., 2009). Displaying the frame can be helpful for students when discussing their learning of the mathematics (see Figure 3).

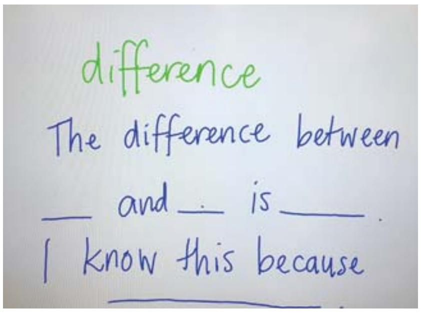

Figure 3. Sentence frames recorded for students

The sentence frames seen in Figure 3 were recorded for students in a Year 1/2 classroom who were playing Diffy Towers for the first time. The frames were designed to support the students' construction of convincing arguments. The first frame required students to name the numbers that were rolled on the dice and then state the difference. The next frame acted as a prompt to articulate ways of thinking and to convince their partner of that thinking.

WAYS OF MODIFYING

Enacting modifications can be achieved by using enabling and extending prompts (Sullivan, Mousley, & Zevenbergen, 2006). I spend more time planning extending prompts because I go into each maths lesson hoping to use them. In rare instances, I have used enabling prompts for Diffy Towers, but these were used only after all students had time to engage with the task first.

ENABLING PROMPT EXAMPLES

- Use Unifix block to make 'mini towers' that represent the two numbers rolled on the dice, place towers side by side, and then snap off the difference to start the Diffy Tower.
- Use modified dice and use only the numbers of 1, 2, 3, and 4 (see Figure 4).

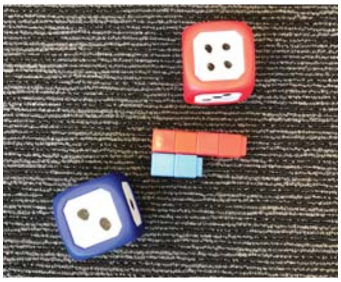

Figure 4. An enabling prompt using modified dice and matching 'mini towers' to see the difference.

EXTENDING PROMPT EXAMPLES

- Use a combination of dice which includes a dot die and a numeral die.
- Use two 10-sided dice to increase the number range.
- Use a combination of a ten-sided die and dot die.
- Use 12 -sided or 20-sided dice to increase the number range (see Figure 5).

Figure 5. Using two 20-sided dice as an extending prompt.

Tasks, like Diffy Towers, have the potential to not only engage students but they can help students understand

© The Mathematical Association of Victoria

important mathematical ideas. Diffy Towers offers the opportunity to extend students' knowledge of subtraction, to explore appropriate counting and reasoning strategies, and to build basic fact fluency.

Diffy Towers holds an important place in my practice, and I hope that this game becomes part of your practice, too. I welcome stories of your use of Diffy Towers with your students.

REFERENCES

Baroody, A.J. (2006). Why children have difficulty mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, *13*(1), 22-31.

Bresser, R., Melanese, K., & Sphar, C. (2009). Equity for language learners. *Teaching Children Mathematics*, *16*(3), 170-177.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Nelson, D. (2008). *Penguin dictionary of mathematics* (4th ed.). London, England: Penguin.

Rowland, T. (2004). The discourse of 'subtraction as difference'. *Proceedings of the British Society for Research into Learning Mathematics*, *24*(2), 91-97.

Schell, L., & Burns, P. (1962). Pupil performance with three types of subtraction problems. *School Science and Mathematics*, *62*(3), 208-214.

Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Developing guidelines for helping students experiencing difficulty in learning mathematics. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces: Proceedings of the 29th Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 496-504). Canberra: MERGA.

MABBLE

Catherine Attard, Western Sydney University
engagingmaths.com
Twitter: @attard_c

A GAME TO ENGAGE STUDENTS WITH MATHEMATICS

We all know children love playing games, but how can we turn this love of games into rich mathematical learning experiences? What are the qualities of a good mathematics game, and should we be incorporating games into regular lessons and homework rather than a Friday afternoon filler activity?

WHY GAMES?

Why use games in the mathematics classroom? First and foremost, they're fun! Of course, that alone isn't a good enough reason to use them. However, when children talk about fun and school, they often perceive fun lessons to be those where they felt challenged and learnt something new. In my research on student engagement, many students talked about fun maths lessons they had experienced, and these are some of their quotes:

> 'Maths is kind of fun when you get to play some maths games' (Year 6)

> '…if you sit on the carpet and the teacher goes on and on about what we're learning it gets boring and you get restless so that's why I like doing fun games.' (Year 6)

> 'Ms. C was a great maths teacher cause she kept giving us different kinds of games that we didn't do before that's about maths. But now it's kind of boring because all we have to do is maths tests, maths stuff, nothing fun about it.'(Year 7)

> 'I loved maths in primary. I remember how we always had these games and we would rotate.' (Year 8)

> 'I like the iPad games because they are really fun and they make me improve on my maths and I like the maths games that tells you when you are wrong or you are right because if you get it wrong you can improve on that'(Year 4).

A good game provides engagement at cognitive, affective, and operative levels. That is, there must be challenge embedded with the game – if it's too easy, children will get bored and no learning will occur. The game must be enjoyable to play, and it must promote interaction and dialogue. There are many mathematics games on the market that are basically drill and practice with the intention of building fluency with number facts.

There are also an infinite number of traditional non-mathematics based games that have a range of mathematical skills and processes embedded within them. The best ones, however, are those that promote the Australian Curriculum: Mathematics proficiencies: Problem Solving, Understanding, Reasoning and Fluency. Take for example, the board game Mabble (http://engagingmaths.co/teaching-resources/mabble-board-game/). Mabble requires an understanding of **place value** and **computation**, but also requires the players to engage in **problem solving** and **reasoning**, while building **fluency** and demonstrating **understanding**. Mabble is self-differentiating; meaning anyone of any ability can play successfully. It is also easy to assess students' work with Mabble as they have to record their work and their scores.

MABBLE GAME PLAY

Mabble is a game suitable for anyone over the age of five. The objective of Mabble is to gain the highest points from completing equations on the game board in a crossword style manner. There are three types of tiles in the game: number tiles, operation tiles and blank tiles. Place all the number tiles face down on the table, and all of the operation tiles face up. The blank tiles can be used to differentiate the game. Each player or pair of players is dealt seven number tiles. The operation tiles remain in a communal pile, for all players to use. Players take turns to use some or all of their number tiles to make one correct equation. Players must build their equations so that they join or intersect either horizontally or vertically with any existing equation that is already on the board. Players replace the number of tiles used after each turn by drawing new number tiles from the pile. For each new equation, players score the total sum of

© The Mathematical Association of Victoria